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Erratum: Neutrino oscillations: quantum mechanics vs. quantum field theory

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The approximation eq. (5.26) is not valid in general, but only if $\Phi_P(E, p)$ and $\Phi_D(E, p)$ do not vary significantly over momentum intervals of order $|p_j - p_k|$, the momentum difference between two neutrino mass eigenstates j and k .

As a consequence, the discussion from the first full paragraph on p. 28 to the last full paragraph on p. 29 should be replaced by:

Let us now discuss the conditions under which eq. (5.25) is a well-defined oscillation probability. For this to take the place, the expression for the differential rate $d\Gamma^{\text{tot}}/dE$ of the overall process should factorize into the production rate, the oscillation probability, and the detection cross section. This means that it should be possible to pull the differential flux and the detection cross section out of the numerator of eq. (5.25) in analogy to eq. (5.6), and then cancel them against the denominator. This, in turn, requires that the momentum distributions $\Phi_P(E, p_j)$ and $\Phi_D(E, p_j)$ be virtually independent of the neutrino mass eigenstate index j . To see when this is the case, we first note that the momentum distribution functions Φ_P are all peaked at the same momentum P . Therefore, if $|p_j - p_k|$ is much smaller than the width of the peak σ_{pP} , we can replace the factors $\Phi_P(E, p_j)$ in eq. (5.25) by the common value $\Phi_P(E, p)$

calculated at the average momentum p , pull them out of the sums in the numerator and in the denominator, and cancel them.¹⁹ A similar argument applies to the momentum distribution functions associated with the detection process, Φ_D , which are all peaked at the same momentum P' and have widths σ_{pD} . When neutrinos are ultra-relativistic or quasi-degenerate in mass, i.e. when

$$|p_j - p_k| \ll p_j, p_k, \quad (\text{E.1})$$

we can also replace p_j, p_k by p in the denominator of eq. (5.25). We can then use unitarity of the leptonic mixing matrix to reduce eq. (5.25) to

$$P_{\alpha\beta}(L, E) = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* e^{i(p_j - p_k)L}. \quad (\text{E.2})$$

Since for ultra-relativistic or quasi-degenerate neutrinos $p_j - p_k \simeq -\Delta m_{jk}^2/2p$, this is just the standard formula for the probability of neutrino oscillations in vacuum. Thus, the QFT-based approach allows one to identify the conditions under which $P_{\alpha\beta}(L, E)$ can be sensibly defined, and also gives the correctly normalized expression for this probability. The conditions are that neutrinos should be ultra-relativistic or quasi-degenerate in mass and, in addition, the inequality

$$|p_j - p_k| \simeq \frac{\Delta m_{jk}^2}{2p} \ll \sigma_p \quad (\text{E.3})$$

should be satisfied. Here, we have introduced the effective momentum uncertainty σ_p , which is dominated by the smallest between σ_{pP} and σ_{pD} . Since σ_{pP} and σ_{pD} , in turn, are dominated by the energy uncertainties σ_{eP} and σ_{eD} , respectively (see section 4.3), condition (E.3) is equivalent to the one in eq. (4.35).

If condition (E.3) is violated, at least one of the momentum distributions $\Phi_P(E, p_j)$, $\Phi_P(E, p_k)$, $\Phi_D(E, p_j)$, $\Phi_D(E, p_k)$ will be strongly suppressed. This implies that the numerator of eq. (5.25) will not factorize in accordance with eq. (5.6) in this case, so that the oscillation probability will not be a well-defined quantity. Also, eq. (5.25) will in general not satisfy the unitarity condition (5.1). It is important that the interference terms involving the suppressed momentum distributions in the numerator of eq. (5.25) will be quenched in this case, and thus neutrino oscillations involving the corresponding mass eigenstates will be inhibited. Physically, this can be traced to the lack of coherence at neutrino production and/or detection. It can be shown that production or detection decoherence is equivalent to the lack of localization of, respectively, the production or detection process [1, 4, 15].²⁰

¹⁹It should be stressed that the mean momentum p is defined here as an average over different mass eigenstates of the momenta $p_j = (E^2 - m_j^2)^{1/2}$ taken at the same fixed value of energy E . It is therefore different from the mean momentum P of the individual wave packets, introduced earlier, for which the average was taken over the spread of momenta (or energies) within the wave packet.

²⁰While condition (E.3) ensures the production/detection coherence (localization), it says nothing about another possible source of decoherence — separation of neutrino wave packets at long enough distances $L > L_{\text{coh}}$ due to the difference of the group velocities of different neutrino mass eigenstates. This is related to the fact that a fixed neutrino energy corresponds to the stationary situation, when the coherence length $L_{\text{coh}} \rightarrow \infty$. The finite coherence length is recovered upon the integration over energy in eq. (5.13) [2].

The reference to eq. (5.26) in the last paragraph of section 5 (p. 30) should be replaced by a reference to eq. (E3) above, and the reference to eq. (5.28) two lines below should be replaced by a reference to eq. (5.7). In addition, the second full paragraph on p. 34 should now read:

We have demonstrated how the QFT approach avoids all the normalization problems of the QM formalism and we have derived the conditions under which it naturally leads to the correctly normalized oscillation probability that automatically satisfies the unitarity condition. The conditions are that (1) neutrinos are ultra-relativistic or quasi-degenerate in mass, and that, in addition, (2) the differences $|p_j - p_k|$ between the momenta of different neutrino mass eigenstates at fixed energy are much smaller than the widths of the neutrino momentum distributions determined by the production and detection processes. If these requirements are not fulfilled, the interaction rate cannot be factorized into the production rate, propagation (oscillation) probability and detection cross section, so that the oscillation probability is undefined. In that case one would have to deal instead with the overall rate of the neutrino production-propagation-detection process.

Finally, the lower right cell of the table 1 becomes:

The oscillation probability $P_{\alpha\beta}(L)$ that is properly normalized and satisfies the unitarity constraint is automatically obtained from the formalism when neutrinos are ultra-relativistic or quasi-degenerate and when, in addition, the momentum differences between different mass eigenstates at fixed energy are much smaller than the widths of the neutrino momentum distributions. Otherwise $P_{\alpha\beta}(L)$ is undefined.